

# FEDERAL PUBLIC SERVICE COMMISSION



## COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

Roll Number

### APPLIED MATHEMATICS, PAPER-I

**TIME ALLOWED: THREE HOURS**

**MAXIMUM MARKS: 100**

- NOTE: (i)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B**. All questions carry equal marks.
- (ii)** Use of Scientific Calculator is allowed.
- (iii)** Extra attempt of any question or any part of the attempted question will not be considered.

### SECTION - A

- Q.1. (a) Find the divergence and curl  $\vec{f}$  If  $\vec{f} = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + (x^2y + 3z^2)\hat{k}$  (10)
- (b) Also find a function  $\varphi$  such that  $\nabla\varphi = \vec{f}$  (10)
- Q.2. (a) Find the volume  $\iint_R xy \, dA$  where R is the region bounded by the line  $y = x - 1$  and the parabola  $y^2 = 2x + 6$ . (10)
- (b) Evaluate the following line integral: (10)
- $\int_c y^2 dx + xdy$  where  $c = c_1$  is the line segment joining the points (-5, -3) to (0, 2), and  $c = c_2$  is the arc of the parabola  $x = 4 - y^2$ .
- Q.3. (a) Three forces P, Q and R act at a point parallel to the sides of a triangle ABC taken in the same order. Show that the magnitude of the resultant is (10)
- $$\sqrt{P^2 + Q^2 + R^2 - 2QR \cos A - 2RP \cos B - 2PQ \cos C}$$
- (b) A hemispherical shell rests on a rough inclined plane whose angle of friction is  $\lambda$ . Show that the inclination of the plane base to the horizontal cannot be greater than  $\arcsin(2 \sin \lambda)$  (10)
- Q.4. (a) A uniform square lamina of side  $2a$  rests in a vertical plane with two of its sides in contact with two smooth pegs distant  $b$  apart and in the same horizontal line. Show that if (10)
- $$\frac{\theta}{\sqrt{2}} < b < a$$
- , a non symmetric position of equilibrium is possible in which  $b(\sin \theta + \cos \theta) = a$
- (b) Find the centre of mass of a semi circular lamina of radius  $a$  whose density varies as the square of the distance from the centre. (10)

## APPLIED MATHEMATICS, PAPER-I

Q.5. (a) Evaluate the integral  $\int_0^1 \int_{x^2}^x (x^2 + y^2) dy dx$  (10)

also show that the order of integration is immaterial.

(b) Find the directional derivative of the function at the point P along z – axis (10)  
 $f(x, y) = 4xz^3 - 3x^2y^2z, P = (2, -1, 2)$

### SECTION – B

Q.6. (a) A particle is moving along the parabola  $x^2 = 4ay$  with constant speed v. Determine the tangential and the normal components of its acceleration when it reaches the point whose abscissa is  $\sqrt{5a}$  (10)

(b) Find the distance travelled and the velocity attained by a particle moving in a straight line at any time t, if it starts from rest at  $t = 0$  and is subject to an acceleration  $t^2 + \sin t + e^t$  (10)

Q.7. (a) A particle moves in the xy – plane under the influence of a force field which is parallel to the axis of y and varies as the distance from x – axis. Show that, if the force is repulsive, the path of the particle supposed not straight and then (10)

$$y = a \cosh nx + b \sinh nx$$

where a and b are constants.

(b) Discuss the motion of a particle moving in a straight line with an acceleration  $x^3$ , where x is the distance of the particle from a fixed point O on the line, if it starts at  $t = 0$  from a point  $x = c$  with the velocity  $c^2 / \sqrt{2}$ . (10)

Q.8. (a) A battleship is steaming ahead with speed V and a gun is mounted on the battleship so as to point straight backwards and is set at angle of elevation  $\alpha$ . If  $v_0$  is the speed of projection (relative to the gun) show that the range is  $\frac{2v_0}{g} \sin \alpha (v_0 \cos \alpha - V)$  (10)

(b) Show that the law of force towards the pole of a particle describing the survey  $r^n = a^n \cos n\theta$  (10)  
is given by  $f = \frac{(n+1)h^2a^{2n}}{r^{2n+3}}$  where h is a constant.

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